MTH 301: Group Theory

Practice Assignment IV

1. Show that every subgroup of a solvable group is solvable.

2. Show that S_4 is solvable.

3. Show that S_n is not solvable for $n \geq 6$. [Hint: Use 1. and 2.]

4. Show that G is solvable if, and only if there exists $N \trianglelefteq G$ such that both N and G/N are solvable.

5. Show that the semi-direct product of two solvable groups is solvable.

6. Show that a group has derived length at most two if and only it has an abelian normal subgroup such that the quotient group is also an abelian group.